TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 4 2019

Calculator Free

Trigonometry

STUDENT'S NAME SOLUTIONS

DATE: Monday 1 July **TIME:** 35 minutes **MARKS:** 34

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, scientific calculator

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (1 mark)

Simplify: $\cos^2\left(\frac{5\pi}{23}\right)+\sin^2\left(\frac{5\pi}{23}\right) = 1$

2. (3 marks)

If $\sin\theta = -\frac{3}{5}$ and $\pi \leq \theta \leq \frac{3\pi}{2}$, find an expression (using fractions) for $\cos 2\theta$

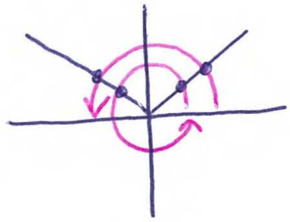
$$\cos 2\theta = 1 - 2\sin^2\theta$$
$$= 1 - 2\left(\frac{-3}{5}\right)^2$$
$$= 1 - 2\left(\frac{9}{25}\right)$$
$$= \frac{7}{25}$$

3. (10 marks)

Solve the following trigonometric equations exactly over the given domains:

(a) $\sqrt{2}\sin(3x) = 1$ $\quad 0 \leq x \leq \pi$ [3]

$\sin(3x) = \frac{1}{\sqrt{2}}$ $\quad 0 \leq 3x \leq 3\pi$



ref: $x = \frac{\pi}{4}$

$3x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$

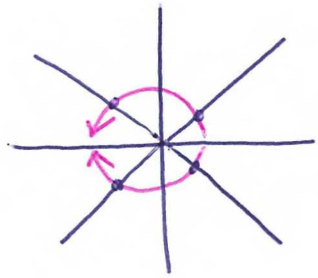
$x = \frac{\pi}{12}, \frac{3\pi}{12}, \frac{9\pi}{12}, \frac{11\pi}{12}$

(b) $3\operatorname{cosec}^2 x - 4 = 0$ $\quad -\pi \leq x \leq \pi$ [3]

$\frac{1}{\sin^2 x} = \frac{4}{3}$

$\sin x = \pm\frac{3}{2}$

$x = \pm\frac{\pi}{3}, \frac{\pm 2\pi}{3}$

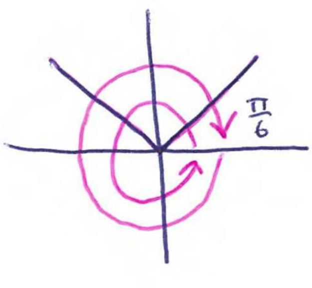


(c) $5\sin x - 2\cos^2 x = 1$ $\quad -2\pi \leq x \leq 2\pi$ [4]

$5\sin x - 2(1-\sin^2 x) - 1 = 0$

$2\sin^2 x + 5\sin x - 3 = 0$

$(2\sin x - 1)(\sin x + 3) = 0$



$\sin x = \frac{1}{2}$ $\quad$ no solution

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{-7\pi}{6}, \frac{-11\pi}{6}$

4. (3 marks)

Prove: $\sin^2\theta\cot^2\theta + 2\sin^2\theta + \cos^2\theta = 2$

$LHS = \sin^2\theta\cot^2\theta + 2\sin^2\theta + \cos^2\theta$

$= \cancel{\sin^2\theta}\dfrac{\cos^2\theta}{\cancel{\sin^2\theta}} + 2\sin^2\theta + \cos^2\theta$

$= 2\sin^2\theta + 2\cos^2\theta$

$= 2(1)$

$= 2$

$= RHS$

QED

5. (4 marks)

Prove the triple angle identity $\sin 3x = 3\sin x - 4\sin^3 x$.

$LHS = \sin(3x)$

$= \sin(2x + x)$

$= \sin 2x\cos x + \cos 2x\sin x$

$= 2\sin x\cos^2 x + (1-2\sin^2 x)\sin x$

$= 2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x$

$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$= 3\sin x - 4\sin^3 x$

$= RHS$

QED

6. (4 marks)

Prove: $(\text{cosec}^2\theta - 2)(\tan^2\theta + 1) = \text{cosec}^2\theta - \sec^2\theta$

$$
\begin{aligned}
\text{LHS} &= (\text{cosec}^2\theta - 2)(\tan^2\theta + 1) \\
&= (\cot^2\theta + 1 - 2)(\tan^2\theta + 1) \\
&= (\cot^2\theta - 1)(\tan^2\theta + 1) \\
&= \cot^2\theta\tan^2\theta - \tan^2\theta + \cot^2\theta - 1 \\
&= \cot^2\theta - \tan^2\theta \\
&= (\text{cosec}^2\theta - 1) - (\sec^2\theta - 1) \\
&= \text{cosec}^2\theta - \sec^2\theta \\
&= \text{RHS} \qquad \text{QED}
\end{aligned}
$$

7. (5 marks)

Prove the following identity.

$1 + 2\cos2\theta + \cos4\theta = 8\cos^4\theta - 4\cos^2\theta$

$$
\begin{aligned}
\text{LHS} &= 1 + 2\cos2\theta + \cos4\theta \\
&= \cancel{1} + 2\cos2\theta + (2\cos^2\theta \cancel{-1}) \\
&= 2\cos2\theta + 2\cos^2\theta \\
&= 2\cos2\theta(1 + \cos2\theta) \\
&= 2(2\cos^2\theta - 1)(\cancel{1} + 2\cos^2\theta \cancel{-1}) \\
&= 4\cos^2\theta(2\cos^2\theta - 1) \\
&= 8\cos^2\theta - 4\cos^2\theta
\end{aligned}
$$

8. (4 marks)

(a) Write the expression $4\sin\theta + 5\cos\theta$ in the form $R\sin(\theta + \alpha)$, where R is a constant and α is an acute angle in radians. [2]

$R\sin(\theta + \alpha) = 4\sin\theta + 5\cos\theta$

$\cos\alpha = 4$

$\cos\alpha = 4/\sqrt{41}$

$\alpha = 0.896$

$\therefore \sqrt{41}\sin(\theta + 0.896)$

(b) Use your expression above to solve algebraically the equation $4sin\theta + 5cos\theta = 4$ for $-\frac{\pi}{2} \leq \theta \frac{3\pi}{2}$. [2]

$\sin(\theta + 0.896) = 4/\sqrt{41}$

ref: $\theta + 0.896 = 0.675, \; 2.467$

$\theta = -0.221, \; 1.571$